Tutorial 10 for MATH 2020A (2024 Fall)

1. Consider the cone frustum $S : z = 2\sqrt{x^2 + y^2}, 2 \leq z \leq 6$. Use a parametrization to express the area of the surface S as a double integral, and evaluate the integral.

2. *The tangent plane at a point* $\mathbf{P_0} = (x(u_0, v_0), y(u_0, v_0), z(u_0, v_0))$ *on a parametrized surface* $S : \mathbf{r}(u, v) = (x(u, v), y(u, v), z(u, v))$ is defined by the plane through $\mathbf{P_0}$ normal to the vector $\mathbf{n} = \mathbf{r}_u(u_0, v_0) \times \mathbf{r}_v(u_0, v_0)$.

   Now consider the circular cylinder $S : \mathbf{r}(\theta, z) = (3 \sin 2\theta, 6 \sin^2 \theta, z), 0 \leq \theta \leq \pi$.

   (a) Find the Cartesian equation for the plane Γ tangent to the surface S at $\mathbf{P_0} = \mathbf{r}(\frac{\pi}{3}, 0)$.

   (b) Sketch the surface S and tangent plane Γ together.

3. Suppose that the parametrized regular(i.e. smooth, non-self-intersecting and with non-vanishing velocity) curve $C : \mathbf{r}(u) = (f(u), g(u), 0), a \leq u \leq b$ lying in xy-plane is revolved about the x-axis, where $g(u) \geq 0$ for $a \leq u \leq b$.

   (a) Sketch the resulting surface S of revolution.

   (b) Find a parametrization of S and represent its surface area as a double integral.

   (c) Calculate the area of the surface S obtianed by revolving the curve $x = y^2, 0 \leq x \leq 1$ about the x-axis.

4. Integrate the function $H(x, y, z) = x^2\sqrt{5 - 4z}$ over the parabolic dome $S : z = 1 - x^2 - y^2, z \geq 0$.

5. Integrate $G(x, y, z) = z - x$ over the portion of the graph of $z = x + y^2$ above the triangle in the xy-plane having vertices $(0, 0, 0)$, $(1, 1, 0)$ and $(0, 1, 0)$.